Nevezetes azonosságok:

$(a+b)^2 = a^2 + 2ab + b^2$ $\qquad (x+3)^2 = x^2 + 6\cdot x + 9$ $\qquad (2x+3y)^2 = 4x^2 + 12xy + 9y^2$

$(a-b)^2 = a^2 - 2ab + b^2$ $\qquad (x-3)^2 = x^2 - 6\cdot x + 9$ $\qquad (2x-3y)^2 = 4x^2 - 12xy + 9y^2$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$ $\qquad (x+y+2)^2 = x^2 + y^2 + 4 + 2xy + 4x + 4y$

$(3x-y+5)^2 = 9x^2 + y^2 + 25 - 6xy + 30x - 10y$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $\qquad (x+2)^3 = x^3 + 3\cdot x^2\cdot 2 + 3\cdot x\cdot 2^2 + 2^3 = x^3 + 6x^2 + 12x + 8$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ $\qquad (x-2)^3 = x^3 - 3\cdot x^2\cdot 2 + 3\cdot x\cdot 2^2 - 2^3 = x^3 - 6x^2 + 12x - 8$

$a^2 - b^2 = (a+b)(a-b)$ $\qquad x^2 - 9 = (x+3)(x-3)$ $\qquad 4x^2 - 36 = (2x)^2 - 6^2 = (2x+6)(2x-6)$

$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$ $\qquad x^3 + 2^3 = (x+2)\left(x^2 - 2x + 4\right)$

$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$ $\qquad x^3 - 2^3 = (x-2)\left(x^2 + 2x + 4\right)$

Gyökvonás azonosságai

$\sqrt[n]{a}\cdot\sqrt[n]{b} = \sqrt[n]{a\cdot b}$ $\qquad \sqrt[3]{x}\cdot\sqrt[3]{y} = \sqrt[3]{x\cdot y}$ $\qquad \sqrt[3]{2}\cdot\sqrt[3]{4} = \sqrt[3]{8} = 2$

visszafelé

$\sqrt[n]{a\cdot b} = \sqrt[n]{a}\cdot\sqrt[n]{b}$ $\qquad \sqrt{25\cdot 3} = \sqrt{25}\cdot\sqrt{3} = 5\cdot\sqrt{3}$

$\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ $\qquad \dfrac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{\dfrac{16}{2}} = \sqrt[3]{8} = 2$

$\sqrt[n]{\sqrt[k]{a}} = \sqrt[n\cdot k]{a}$ $\qquad \sqrt[5]{\sqrt[3]{x}} = \sqrt[15]{x}$

$\sqrt[n]{a^k} = a^{\frac{k}{n}}$ $\qquad \sqrt[5]{x^3} = x^{\frac{3}{5}}$ $\qquad \sqrt[3]{2^8} = 2^{\frac{8}{3}}$

$\overline{\overline{A}} = A$	
$\overline{\emptyset} = H$	$\overline{H} = \emptyset$
$A \cup \emptyset = A$	$A \cap H = A$
$A \cup H = H$	$A \cap \emptyset = \emptyset$
$A \cup \overline{A} = H$	$A \cap \overline{A} = \emptyset$
$A \cup B = B \cup A$	$A \cap B = B \cap A$
$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
$A \cup A = A$	$A \cap A = A$
$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$

3.4.4. Gyökök azonosságai

$\forall a,\ b \in \mathbb{R}^+;\ \forall n,\ k \in \mathbb{Z};\ n, k \geq 2.$ $\sqrt[n]{a} = b \iff b^n = a.$

Racionális kitevőjű hatvány: $a^{\frac{1}{k}} := \sqrt[k]{a};\ a^{\frac{n}{k}} := \sqrt[k]{a^n}.$

Azonos alapú gyökök: (lásd 10.3. táblázat, 80. oldal)

$\sqrt[n]{a} \cdot \sqrt[k]{a} = \sqrt[nk]{a^{n+k}}.$ $\sqrt[n]{a} : \sqrt[k]{a} = \sqrt[nk]{a^{k-n}}.$

$\sqrt[k]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[k]{a}} = \sqrt[nk]{a}.$ $\sqrt[k]{a^n} = \left(\sqrt[k]{a}\right)^n = a^{\frac{n}{k}}.$

Azonos kitevőjű gyökök:

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$ $\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{a : b}.$

3.4.5. Logaritmusok azonosságai (lásd 10.4. táblázat, 84. oldal)

$\forall a, b, c \in \mathbb{R}^+ \setminus \{1\},\ \forall x, y \in \mathbb{R}^+,\ \forall n \in \mathbb{Z}.$ $a^{\log_a b} = b.$

$\log_a b = c \iff a^c = b.$

Azonos alapú logaritmusok:

$\log_a(x \cdot y) = \log_a x + \log_a y.$ $\log_a(x : y) = \log_a x - \log_a y.$

$\log_a(x^n) = n \cdot \log_a x.$ $\log_a\left(\sqrt[n]{x}\right) = \left(\log_a x\right) : n.$

$\log_a a = 1.$ $\log_a 1 = 0.$

Különböző alapú logaritmusok:

$\log_a b \cdot \log_b a = 1.$ $\log_b x = \dfrac{\log_a x}{\log_a b} = \log_b a \cdot \log_a x.$

$\lg x = \log_{10} x.$ $\ln x = \log_e x.$

$\lg x = M \cdot \ln x \approx 0{,}43429 \cdot \ln x.$ $\ln x = \dfrac{1}{M} \lg x \approx 2{,}3059 \cdot \lg x.$

I. SZÁMOK, MŰVELETEK 1.

1. Hatványozás azonosságai

1. Számítsuk ki az alábbi hatványokat:

a) 2^2

b) 3^3

c) $\left(\frac{1}{3}\right)^{-3}$

d) $(-1)^2$

e) $\left(\frac{1}{2}\right)^3$

f) 4^{-1}

g) 2^{-2}

h) $\left(-\frac{1}{3}\right)^3$

i) $(-1)^{-1}$

megoldás:

a) $2^2 = 2 \cdot 2 = 4$

b) $3^3 = 3 \cdot 3 \cdot 3 = 27$

c) $\left(\frac{1}{3}\right)^{-3} = 3^3 = 27$

d) $(-1)^2 = (-1) \cdot (-1) = 1$

e) $\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) = \frac{1}{8}$

f) $4^{-1} = \frac{1}{4^1} = \frac{1}{4}$

g) $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

h) $\left(-\frac{1}{3}\right)^3 = -\frac{1}{3} \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) = -\frac{1}{27}$

i) $(-1)^{-1} = \frac{1}{(-1)^1} = -1$

2. A hatványozás azonosságainak felhasználásával végezzük el az alábbi műveleteket:

a) $x^3 \cdot x^{-5}$

b) $x^2 \cdot x^7 \cdot x^5$

c) $\frac{x^2 \cdot x}{x^9}$

d) $\frac{a^2}{a}$

e) $\frac{a^5}{a^6}$

f) $x^2 \cdot x^{-2}$

g) $(a^2)^3$

h) $\frac{a^3 \cdot a^4}{a^2}$

i) $\frac{a^2 \cdot a^4}{a}$

megoldás:

a) $x^3 \cdot x^{-5} = x^{3+(-5)} = x^{-2} = \frac{1}{x^2}$

b) $x^2 \cdot x^7 \cdot x^5 = x^{2+7+5} = x^{14}$

c) $\frac{x^2 \cdot x}{x^9} = \frac{x^3}{x^9} = \frac{1}{x^6}$

d) $\frac{a^2}{a} = a^{2-1} = a$

e) $\frac{a^5}{a^6} = a^{-1} = \frac{1}{a}$

f) $x^2 \cdot x^{-2} = x^{2-2} = x^0 = 1$

g) $(a^2)^3 = a^{2 \cdot 3} = a^6$

h) $\frac{a^3 \cdot a^4}{a^2} = \frac{a^{3+4}}{a^2} = \frac{a^7}{a^2} = a^5$

i) $\frac{a^2 \cdot a^4}{a} = \frac{a^{2+4}}{a} = \frac{a^6}{a} = a^5$